LINEARITY AND NONLINEARITY IN HYDROLOGIC RESPONSE IN ARID AND SEMIARID WATERSHEDS

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ABSTRACT: The issue of linearity and nonlinearity of watershed response has been raised by hydrological researchers in relation to watershed scale. Hydrologic response is usually assumed to approach linearity with increasing watershed scale. To investigate the validity of the assumption in arid and semiarid environments, a lumped model based on the Nash cascade of linear reservoirs was applied to a number of sub-watersheds in the semiarid Walnut Gulch Experimental Watershed in southern Arizona. By using the Nash cascade of linear reservoirs, the rainfall-runoff relation was modelled. The Nash model was parameterized by the method of moments. Net rainfall was calculated by using three loss models. The rainfall-runoff model was found to be applicable to watersheds at particular scales. Results showed that as the watershed scale decreased, the hydrologic response approached linearity. In arid and semiarid watersheds, variability in space and time is intrinsic in the convective storm events which yield runoff that may even vanish through infiltration in the channel bed before reaching the watershed outlet. Two main sources of nonlinearity might be transmission losses and spatial variability of rainfall. In arid and semiarid watersheds, nonlinearity of watershed response in larger areas might require a distributed modelling approach at a warranted complexity level. The model should allow for spatial variability of rainfall and provide for transmission losses effects on the hydrograph.

KEYWORDS: Hydrologic response, Rainfall-runoff modelling, Lumped model, Linear reservoir, Nash model, Arid regions, Semiarid regions, Watershed scale

1. INTRODUCTION

Hydrologic response is usually assumed to approach linearity with increasing watershed scale [28]. The issue of linearity and nonlinearity of watershed response has been raised by hydrological researchers in literature [28, 25, 9]. In more than on publication the issue was discussed in relation to semiarid zones. Minshal [16] and Wang et al [30] showed that nonlinearity in watershed response decreased with increasing watershed area in humid regions. Wang et al [30] studied nonlinearity in watershed response in humid zones using a transfer function. Goodrich et al [9] approached the problem by invoking geomorphological models and statistical properties of watersheds. Goodrich et al [9] showed that linearity of watershed response increased with decreasing watershed area in arid and semiarid regions, which Sivapalan et al [28] suggested to be defined as scaling relationship. In contrast, they suggest that the conventional definition for nonlinearity be kept for the dynamic relationship in rainfall-runoff response. Vivoni et al [29] discussed nonlinearities in the rainfall-runoff transformation and its scale-dependence.

Runoff hydrographs of single events are characterised by the steep, almost perpendicular, rising limb of the hydrograph contrasted with a longer gradually fading falling limb. Therefore, time to peak flow is very short relative to storm duration. It may be in the order of a few minutes. It simply reflects the
high intensity of rainfall events, thus dictating short time steps for simulation, so that peak flow is not missed. Simulating event hydrographs entails accounting for losses in the way they progress during the event. However, that is very difficult to represent in a working form. Another dimension of the difficulty is the size of the watershed where a model can be used. Cantón et al [2] provided an overview of the key factors and processes that influence runoff generation in semiarid environments including watershed scale.

Modelling response to rainfall stimulus in arid and semiarid watersheds is generally elusive. The usual response is in the form of flash floods, which are initiated by localised high-intensity-short-duration rainfall storm events. Variability in space and time is intrinsic in the convective storm events which yield runoff that may even vanish through infiltration in the channel bed before reaching the watershed outlet [32]. Flow in the channels is ephemeral, leaving the bed dry for prolonged periods. For a given event, the whole watershed may not be activated, and that is mostly the case. At the event scale, infiltration losses are always substantial relative to yield volume. Initial and transmission losses (continuing abstractions along the path of the flood wave) are two significant processes in such watersheds. Owing to these losses, only a small fraction of rainfall is transformed into runoff.

Complex models have been tested in semiarid regions and did not prove to be significantly superior to simpler ones, considering the data, parameter and computational requirements [14, 4, 5]. But the question remains as to what is the appropriate watershed size that a model can represent and simulate the hydrograph. Thus, simple conceptual models should be tested in a complex environment. Then possibly a watershed size, or a maximum size, may be defined such that the lumped model works. Elaborating a model by parameterisation may require further calibration.

Another advantage is the ability of a conceptual model to describe a group of physical processes in a simplified approximate manner, yet it establishes a cause-and-effect relationship between input and output. Thus, a degree of empiricism is also involved, which is justified by presently missing knowledge on a given physical attribute of a watershed. By studying a range of values of a given parameter in a conceptual model, one can come to a generalisation that a given model is applicable to conditions that prevail when such a range is replicated.

One widely-used conceptual watershed model, a Nash cascade of linear reservoirs [27], is based on the concept of linking a number, \( n \), of linear reservoirs in series, whereby the outflow from the first is the inflow to the second and the outflow from the second is the inflow to the third and so on. Finally, the outflow from the last reservoir is the watershed outflow. A linear reservoir is an element whose outflow is directly proportional to storage. Principally, a linear reservoir provides a certain amount of attenuation with an associated lag. Subsequently, each reservoir in the series is assumed to have the same lag, \( k \), and provides the same amount of attenuation. A cascade of linear reservoirs is a conceptual model with two parameters, \( n \) and \( k \). \( n \) is the number of linear reservoirs that a given volume of water has been routed through and \( k \) the time scale (time lag) of each reservoir; thus \( nk \) is the watershed lag. \( n \) and \( k \) are estimated by the method of moments. The analytical solution of the cascade of linear reservoirs is the Nash model, which has been used in this analysis.

The objective of this paper is to establish a relationship between watershed size and the dynamic relationship between rainfall and runoff in semiarid regions by using a transfer function, namely the IUH, as derived by Nash [18, 19] for a cascade of linear reservoirs. In this paper analysis of the characteristics of runoff hydrographs in a semiarid watershed was performed. By using the Nash cascade of linear reservoirs, the rainfall-runoff relation was modelled. The Nash model was parameterised by the method of moments. Net rainfall was calculated by using three loss models, for the sake of comparison.
The model was applied at the Walnut Gulch Experimental Watershed in Arizona, USA. The model was applied in the main watershed (WG01) and in three of its sub-watersheds (WG11, and WG04, WG111). The model was found to be applicable to watersheds at particular scales. Results showed that as the watershed scale decreased, the hydrologic response approached linearity. Sources of nonlinearity were identified.

2. MATHEMATICAL MODELLING

2.1 The Cascade of Linear Reservoirs

Representing the watershed by an activated reservoir series is the underlying assumption in conceptualising the response of the watershed to rainfall stimulation. For the case at hand, the Nash cascade of linear reservoirs is adopted to describe the rainfall-runoff relation.

A linear reservoir has a storage that is related to its output linearly by a storage constant (time scale, lag time) \( K \)

\[
S = KQ
\]

(1)

where:
- \( S \): the storage term
- \( K \): is the storage constant (time scale, lag time)
- \( Q \): is discharge

\[
\frac{dS}{dt} = I - Q
\]

(2)

Substituting for \( S \) from Equation 1 and integrating Equation 2 gives,

\[
\ln(I - Q) = -\frac{t}{k} + c
\]

(3)

When \( S(0)=1 \) and \( I(t) =0 \) for \( t>0 \), we obtain IUH denoted by \( u(0,t) \), which is the impulse response function for a linear reservoir,

\[
u(0,t) = \frac{1}{k} e^{-\frac{t}{k}}
\]

(4)

A watershed can be represented by an \( n \)-series of identical linear reservoirs that have the same time scale \( k \); that is to say a cascade of linear reservoirs, also known as the Nash Cascade. When a unit volume of water is routed through the \( n \)-linear reservoirs, the instantaneous unit hydrograph (IUH) can be derived [6].

Discharge from the first reservoir in the cascade would be the inflow to the following and so on. By applying the convolution integral to Equation 4, the outflow of the 1-th reservoirs is given by:

\[
q_1 = \int_0^t \frac{1}{k} e^{-\frac{t}{k}} dt
\]

(5)
If the time since the beginning of an event until the input \( q(t) \) began is estimated and \( q_1 \) is the input function, then the outflow from the second reservoir is defined, and so on. Convoluting successively through \( n \) reservoirs, or, alternatively, routing the unit volume through \( n \) reservoirs the IUH \( (q_n) \) for the cascade is obtained as:

\[
q_n = \frac{1}{\Gamma(n)k} \left( \frac{t}{k} \right)^{(n-1)} e^{-\frac{t}{k}}
\]

Equation 6 has two parameters; \( k \) and \( n \). \( \Gamma(n) \) is the gamma function. In other words, the IUH is a gamma distribution function. To estimate the parameters \( n \) and \( k \), first and second moments of net rainfall and discharge about the origin are computed.

### 2.2 Estimating the Parameters \( n \) and \( k \)

As mentioned above, a Nash cascade is composed of a series of similar linear reservoirs that can assume a value of any real number \( (n) \), with the same lag time \( (k) \). The lag time (time scale) for the whole watershed, designated by \( K \) in Equation 1, is equivalent to the product term \( (nk) \). In this case \( nk \) is the first moment of the IUH about the origin. In discrete time intervals, the first and the second moments of the IUH (a unit volume of flow) about the origin are as follows [6, 17]:

\[
MIUH_1 = nk \quad (7)
\]

\[
MIUH_2 = n(n+1)k^2 \quad (8)
\]

Further it can be shown that:

\[
MQ_1 - MI_1 = nk \quad (9)
\]

\[
MQ_2 - MI_2 = n(n+1)k^2 + 2nk MI_1 \quad (10)
\]

where:
- \( MI_1 \): first moment of net rainfall hyetograph about the origin
- \( MI_2 \): second moment of net rainfall hyetograph about the origin
- \( MQ_1 \): first moment of runoff hydrograph about the origin
- \( MQ_2 \): second moment of runoff hydrograph about the origin

The values of \( MI_1, MI_2, MQ_1, \) and \( MQ_2 \) were estimated from the rules of mechanics by using the discrete time interval runoff hydrographs and hyetographs. By substituting the estimated moments in Equations 7 and 8 the parameters \( n \) and \( k \) could be calculated. The product \( nk \) is the difference of moments that is equal to the distance between the centroids of rainfall hyetograph and runoff hydrograph. After \( n \) and \( k \) were determined, the IUH was constructed by applying Equation 6. Further, the IUH was integrated to determine the TUH. Finally, by convolution of the IUH and the effective rainfall the runoff hydrograph was calculated using the following convolution integral.

\[
Q(t) = \int_0^t I(t)q(t-\tau)d\tau \quad (11)
\]

\( I(t) \) is the effective rainfall.
2.3 Modeling Approach

2.3.1 Lumped Linear Model

The principal advantage of lumped modelling is the minimal data requirement. It is easier to estimate the parameters, but then accuracy of representation might be sacrificed. In semiarid watersheds, spatial and temporal variability of rainfall is always important. Heterogeneity of watershed characteristics has also a significant effect on hydrologic parameters [1, 32]. Nonetheless, lumped modelling should not be excluded; rather it should be investigated to define the limitations.

2.3.2 Modelling Assumptions

It was assumed that the response of the watershed to the rainfall stimulation was linear. A Nash cascade was formulated for the watershed by assuming that the lag time \( t_{nk} \) was the difference between the centroid of effective rainfall and the centroid of runoff hydrograph. The effective rainfall volume was found by equating rainfall volume to the total runoff volume, after subtracting applicable losses. Effective rainfall distribution depended on the assumed loss model as discussed in section 2.3.3.

2.3.3 Loss Models

Calculation of losses relied on balancing effective rainfall volume with discharge volume. Initial losses were calibrated to yield effective rainfall volumes equal to discharge volumes. Effective rainfall was calculated by invoking three loss models: a constant loss model, proportional loss model and a continuing loss model after subtracting an initial loss [21, 13]. Proportional losses were estimated by a volumetric runoff coefficient. Proportional loss model accounts also for the translation as the rainfall starts before the initiation of the hydrograph.

In the proportional loss model one uses a runoff coefficient that is calculated as the ratio of input and output. As the intensity of rainfall was assumed constant during each time interval, according to this loss model runoff must be produced from each interval. Nonetheless, the proportional loss model did not err by much as compared to the continuing loss model.

Effective rainfall did not always result from the highest intensities in the storm. Nonetheless, for the constant loss model, the high intensities would generate runoff. In the case of the other two loss models, the net rainfall was always the volume left after subtracting initial losses.

3. CASE STUDY WATERSHED

Located in south eastern Arizona, USA, the area of Walnut Gulch Experimental Watershed is about 148 km². It is approximately located around 110° W, 31°45’N. Topography is comprised of rolling hills and some steep terrain, ranging in elevation between 1190 and 2150 m+msl. Cattle grazing and recreational activities are the major land uses, although some urbanization exists in and around the town of Tombstone. Vegetation within the watershed is composed primarily of grassland and shrub-
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steppe rangeland vegetation. The geology is dominated by a thick alluvial fan (about 400 m in depth) contributing to the San Pedro River with abundant groundwater at levels ranging from a few meters to 100m in depth [23, 24].

Stream channels within the Watershed have been recently incised and are ephemeral [12]. Rainfall is bimodal; a summer and a winter mode. The latter mode is a result of frontal storms in the winter months. These frontal storms have low intensities (< 25 mm/hr) and do not usually cause runoff [10]. However, most runoff events occur as a result of high intensity, short duration summer storms. In the summer the storms are convective and are limited spatially with intensities higher than 25 mm/hr [15, 26]. With an average annual rainfall of 324 mm and an annual mean temperature of 17.6 °C in the city of Tombstone, the area’s climate is classified as semiarid [24].

4. IMPLEMENTING THE MODEL

4.1 Data Preparation

The original records had values of rainfall or runoff at irregular intervals. Rainfall and runoff records were interpolated to produce fixed-time-interval records for each storm. The events were interpolated using time intervals starting at one minute and increasing by unity up till five minutes. The reason for small time intervals was to make sure that the peak flow was not missed. The model was tested for all five time intervals for accuracy and it was proved to be insensitive to the time intervals.

Isohyetal maps of storm events were prepared using GIS. The purpose was to analyze the spatial variations to help in storm selection (See Figure 1 and Figure 2). By investigating the isohyetal maps it can be seen that there is a pattern that defines the shape of the hydrograph, depending on the location of the storm. The storm distribution and the size of storm peak determine the size and the time of peak flow. Since storms in arid and semiarid watersheds are localized, it was difficult to find storms that covered all considered sub-watersheds simultaneously.

Data files were prepared for the whole Walnut Gulch Watershed draining at Flume 1 (WG01) (148 km²) and for three of its sub-watersheds draining into Flume 4 (WG04) (2.3 km²), Flume 11 (WG11) (8.4 km²) and Flume 111 (WG111) (0.6 km²). The number of storms analysed was 29 for WG01, 25 for WG04 and 26 for WG11 and 12 for WG111. WG111 has a shorter measurement record than the other ones.

![Fig. 1: Rainfall depth isohyets for storm 27/08/1982](image)
4.2 Criteria for Storm Selection

Criteria of storm selection were dictated by the pattern of rainfall and runoff in the case study watershed. It was necessary to select standard events that represent the usual hydrologic response of the watershed. An example of a standard event is shown in Figure 3. These criteria are outlined as follows.

- mostly individual peak hydrograph structure,
- continuous distribution of rainfall over the time span of the storm with discontinuities in some cases,
- direct runoff peak were in the order of a few cubic meters per second, so that the flow is mainly on the surface

4.3 Coding the Model

A computer programme to apply the Nash linear model procedure was prepared by using MATLAB. A script and a user interface for input and output were prepared for the model. Output was in the form of graphs and data files.

The model computer code calculated losses from total rainfall to produce effective rainfall. The value of effective rainfall was calibrated to the volume of discharge by trial and error. Trial and error attempts were based on fitting the calculated and observed hydrographs. By running the model several times the calculated hydrograph was fitted to the observed one. Three error measures were used. Root mean square error (RMSE), mean absolute error (MAE) and the coefficient of efficiency (CE) [20]. The formulas for the three measures are as follows.

\[
RMSE = \left(\frac{(O_i - P_i)^2}{n}\right)^{0.5}
\]

\[
MAE = \frac{|O_i - P_i|}{n}
\]

\[
CE = 1 - \frac{(O_i - P_i)^2}{(O - \bar{O})^2}
\]

where:
- \(O\): observed
- \(P\): calculated
- \(\bar{O}\): average observed values
- \(i\): index
5. DISCUSSION OF RESULTS

5.1 Model Performance with Respect to Scale

Acceptable model results were obtained in the smaller sub-watersheds. While in the main watershed the model was for the most part unsuccessful. Unacceptable results were possibly due to unexplained routing effects and the assumption that hydrologic response was linear. Model output for some events is shown in a Table 1 for each watershed area.

Characteristics of the hydrograph of the main watershed differed slightly from those in the sub-watersheds. However, it seems that the model performed better in the sub-watersheds. Rainfall uniformity might be higher in the smaller areas as shown by the uniformity coefficients (Figure 4). The uniformity coefficient (CU) was calculated according to Christiansen [7] as follows.

\[ CU = \frac{\sum |P_v - P_{avg}|}{\sum P_v} \]  

(4)

\( P_v \): raingage depth,
\( P_{avg} \): average rainfall depth,
\( r \): index

Such uniformity and scale effects might be the reason for minimising the errors of averaging and lumping in small scales. The coefficient of efficiency was higher in the smaller areas (Figure 5). Obviously, the linear response assumption at small scale might be valid as the model has demonstrated better results at that scale.
Table 1: Linear model output

<table>
<thead>
<tr>
<th>Event Date (yymmdd)</th>
<th>Rainfall Intensity (mm/hr)</th>
<th>Average Q (mm/hr)</th>
<th>nk (min)</th>
<th>Observed Peak Flow (m³/sec)</th>
<th>Calculated Peak Flow (m³/sec)</th>
<th>Time to Peak</th>
<th>Initial Loss (mm/hr)</th>
<th>MAE (m³/sec)</th>
<th>RMSE (m³/sec)</th>
<th>Coefficient of Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear model output for WG01 (148 km²)</td>
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<tr>
<td>900801</td>
<td>3.3</td>
<td>0.1</td>
<td>112</td>
<td>12.7</td>
<td>10.1</td>
<td>13</td>
<td>24</td>
<td>0.2</td>
<td>0.7</td>
<td>0.91</td>
</tr>
<tr>
<td>910827</td>
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<td>0.1</td>
<td>70</td>
<td>19.0</td>
<td>12.7</td>
<td>13</td>
<td>12</td>
<td>0.2</td>
<td>1.1</td>
<td>0.91</td>
</tr>
<tr>
<td>Linear model output (WG11) (8.4 km²)</td>
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<tr>
<td>940829</td>
<td>5.8</td>
<td>0.4</td>
<td>17</td>
<td>3.9</td>
<td>3.6</td>
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<td>0.22</td>
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<tr>
<td>960710</td>
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<td>5.6</td>
<td>8</td>
<td>55</td>
<td>0.04</td>
<td>0.23</td>
<td>0.96</td>
</tr>
<tr>
<td>Linear model output (WG04) (2.3 km²)</td>
<td></td>
<td></td>
<td></td>
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<td>1.8</td>
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<td>1.5</td>
<td>11</td>
<td>41</td>
<td>0.01</td>
<td>0.06</td>
<td>0.98</td>
</tr>
<tr>
<td>Linear model output (WG111) (0.6 km²)</td>
<td></td>
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<td>0</td>
<td>130</td>
<td>0.03</td>
<td>0.19</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Note: MAE : mean absolute error  RMSE : root mean square error

In the small watershed areas, the method of moments might be more suitable to single peak events that result from high intensity rainfall events. It is limited by the temporal and spatial distribution of rainfall intensity [3]. Thus, the cases of high intensity short duration were more suitable to the method. In the case when the method failed it might be due to location of effective rainfall hyetograph, which might intersect with runoff hydrograph. Zoccatelli et al [33] indicated the influence of the spatial rainfall variability on runoff modelling for a small catchment size.

In some cases the total volume of the hydrograph was reasonably matched with the observed one in the large area. However, the peak and the time to peak did not match well in the large watershed that might be due to the variability of rainfall. It might be that the variability of rainfall limits the applicability of the method of moments. The method of moments calculates the watershed lag from an average rainfall hyetograph. This hyetograph is averaged over a large area, which is not necessarily activated by the storm. On the contrary in the small areas more rainfall uniformity was observed, leading to better results (Figure 6).

Calculation of losses in all areas assumed only infiltration losses. The loss models did not provide for transmission losses. In the case of the small areas these losses were not significant. Transmission losses are more significant at the large watersheds (See Figure 3). Although sub-watershed WG11 was not very large, high transmission losses were observed there [9]. The runoff hydrograph in a large area travels a considerable distance and loses water to channel infiltration. The linear Nash model routes the hydrographs without considering transmission losses. That might be the other reason for the better performance of the model in small areas.
The transmission losses have less effect on the response in an arid catchment as the size diminishes [11]. Variability of rainfall may be present even at the smaller catchment scale in arid zones, although lessened by the size of the rain cells [31, 1, 32].

Fig. 4: Uniformity coefficients

Fig. 5: Coefficients of efficiency
5.2 Peak Flow

In the successful cases in the main watershed area, the calculated hydrographs were dominated by underestimated peak flow and the lag in response. The underestimation of peak might be ascribed to the routing effect of the linear reservoir [22]. A linear reservoir translates and attenuates the hydrographs during routing. Hydrographs of selected events at the modelled watershed areas are shown in Figure 7 through Figure 10. Comparison of calculated and observed peak flows at all watershed areas are shown in Figure 11 through Figure 14.

Fig. 7: Calculated and observed hydrographs

Fig. 8: Calculated and observed hydrographs

Fig. 9: Calculated and observed hydrographs

Fig. 10: Calculated and observed hydrographs
In general, the peak flow values were more accurately calculated in the smaller areas. In the case of WG04 (2.3 km²), the calculated matched observed peaks more accurately than the other cases. The assumption of linearity might hold at a certain size of watershed area. Alternatively, it may be fairly accurate such that it reciprocates little error for large simplicity.

5.3 Time to Peak

In the small watersheds the time to peak was simulated with greater accuracy than the peak flow. For the model to compensate for simulating the rising limb linearly, continuity was satisfied by deducting mass from the peak flow. Physically the watershed storage does not build up gradually; it is rather augmented in a short time by the flash flood. The magnitude of the “short time” is in the order of a few minutes. As mentioned earlier the storms were usually localized. In the case of the large areas the time of travel of the hydrograph would be longer, thus the routing effects and the transmission losses would be more significant. The flow wave gets abstracted at a high rate as transmission losses at the beginning of flow. The model did not provide for this process. In addition the routing effects might also be the reason for the symmetrical calculated hydrographs in WG01.

In Figure 7 the uniform rising limb of the calculated hydrograph at WG01 almost matches the uniform decline. In arid watersheds, most storm events produce the sudden steep rising limb and the gradual decay of the falling limb. The linear reservoir effect is clear on the hydrograph.

5.4 Watershed Lag Time

The values of k and n resulted from the assumptions of the loss calculations. So, the k and n values would reflect all the errors in assumptions. In some cases, when the rainfall continued beyond the hydrograph time base, the calculations resulted in effective rainfall that occurred after the centroid of the hydrograph. In turn that resulted in negative net moment (thus negative values of nk). In other words, MI₁ and MI₂ exceeded MQ₁ and MQ₂. In that case a negative watershed lag is calculated which is not acceptable. This is one limitation in the method of moments. It might be worthwhile to analyse the differences when k and n are adjusted by trial and error calibration. Such approach might be feasible with single peak storms.

![Calculated Peak Flow (Qcal) vs Observed Peak Flow (Qobs) (WG01)](image1)

![Calculated Peak Flow (Qcal) vs Observed Peak Flow (Qobs) (WG04)](image2)

**Fig. 11:** Calculated versus observed peaks  
**Fig. 13:** Calculated versus observed peaks
For some storms in the large areas results were not reasonable. The method of moments calculates the difference in time between the occurrence of the centroids of the effective rainfall hyetograph and the hydrograph. For the method to yield reasonable results the centroid of rainfall must occur prior to the centroid of runoff. It describes a cause-and-effect relationship, which is in accordance with the assumptions of the linear system theory. In some cases the difference of time/moments was negative, which is not in accordance with the model assumptions. That outcome might be due to possible errors in data and to errors in associating rainfall-runoff events. It seems that parameter estimation by the method of moments was sensitive to synchronization errors. In general the variations in watershed lag time value with peak flow are lower in the smaller watersheds, WG11, WG04 and WG111. The watershed lag time ($n_k$) is plotted versus the peak flows as shown in Figure 15 through Figure 18. A declining trend is more apparent in WG01 than the other three smaller areas. It might also imply that linearity might be assumed at smaller scales.
5.5 Uncertainty in Estimation of Losses

Regarding loss models, none of them proved to be better in improving the fit of the hydrograph. In most cases, the continuing loss model yielded slightly better results than the other two and mostly when the initial loss rate was more than the rainfall intensity. It is clear that the losses were high, which is expected in such cases. It may not be always true that the highest intensity of rainfall would cause the runoff. The soil profile may have not reached its saturated conditions at that point in time. At that high intensity, the soil infiltration rate may not have been exceeded.

In the case of the proportional loss model only some storms in WG01 yielded usable results. The rest did not work due to the extended duration of the rainfall relative to the runoff.

Transmission losses were not provided for in the invoked loss models. As shown in the example in Figure 3, and in many other cases, it is clear that the hydrograph was almost halved during transmission between the three stations, although the area between the two stations received part of the storm.

None of the loss models was significantly superior to the other in the case of the large area. The similarity in performance might be explained by the short duration of storms. The distribution of losses over a short time span did not change the difference in the second moments of effective rainfall hyetograph and runoff hydrograph by an apparent magnitude. This was valid for the successful cases, while for the cases that did not succeed; the events might have produced non-physical negative moment difference.

5.6 Classification of Model Output

Responses can be assembled in three groups, depending on the value of the watershed lag parameter: \( n_k \). In one group one finds the relatively high value \( n_k \), exceeding 200 min. In the other comes the group with \( n_k \) at values less than a 100, and at the third the group of values at about 50.

For each group, certain traits are observed. In the first group the simulated hydrograph barely fits the observed one. The reasons might be the excessive second moment (MI2 and MQ2) that leads to negative values of \( n \). In other words, moment variations are magnified by the square of the second moment, which arise from large storm lag. While the second group comes with a lower \( n_k \) value, it calculates a low peak flow as compared to the third one. The peak is mostly more than 30% in error of the measured one. The third group was close to simulating the hydrograph, with a value of \( n_k \) less than 100. In the case of the double peak, it was invariably not possible to fit both because of the break of rainfall within events that lead to runoff; in other words; in the case of non-continuous storms. In the case of storms with two rainfall peaks, the simulated hydrograph was relatively close to the measured one.

6. CONCLUSION

The watershed was modelled as a cascade of linear reservoirs using the Nash approach. The single-event modelling approach is used to model the hydrograph of flash floods in Walnut Gulch Experimental Watershed. The model has been used for the Walnut Gulch main watershed and some of the sub-watersheds. Applicability of the model at the smaller scale watersheds was found more accurate. The main finding was that the model performed better in the small areas of the watershed rather than the large areas. It may indicate that linearity increases at smaller watershed sizes.

Furthermore, the variations in the spatial distribution of rainfall are sufficiently extreme. Lumping might lead to deleting the effect of rain peaks. Runoff producing areas are different for every storm, due to
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spatial variability of rainfall. Transmission losses reduced the volume of the hydrograph in the large basins. Based on those observations in arid and semiarid regions, linearity of response might be assumed at smaller basins, rather than large basins.

Peak flow value and time were difficult to simulate. In the large area, error in the time to peak was not less than the error of magnitude. The start of the hydrograph rising limbs did not match in most cases even with the initial-continuing loss model.

Both peak flow and the shape of the hydrograph are related to the location of the raincells within the storm event upstream. It was observed that the location of the storm in the watershed influenced the shape of the hydrograph. In other words, the distribution of rainfall has an effect on the timing and shape of the hydrograph in the large watershed. In the smaller areas the rainfall distribution was sufficiently uniform not to affect the hydrograph.

Errors were also caused by the underlying lumped linear assumptions of watershed response. It is assumed in the linear theory that every impulse must have a response. Net rainfall was the impulse to cause the response. In the calculation of losses it was assumed that the highest intensities were the cause of runoff. It might not be the case always, especially in the large areas. Furthermore, in arid watersheds that might not be the case since initial and transmission losses have different effects on runoff generation. In this model, losses are considered as one single process. On the contrary transmission losses occur in the channel bed after initial infiltration losses are satisfied and runoff has accumulated.

Two sources of nonlinearity might be identified as transmission losses and spatial variability of rainfall. When routing the hydrographs, transmission losses should be accounted for in arid and semiarid watersheds. Rainfall spatial variability is significant in such areas. All these processes might be the causes of nonlinearity in watershed response. Modelling an arid watershed by the linear lumped approach may not properly describe the watershed response. Therefore, a distributed approach may be the viable alternative, at a warranted complexity level. An alternative modelling approach is required. The model should provide for transmission losses effects on the hydrograph, allow for spatial distribution of rainfall input and route excess rainfall non-linearly.

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8. REFERENCES


